The Q-elasticity of Investment*

Boyan Jovanovic and Peter L. Rousseau

September 12, 2007

Abstract

(PRELIMINARY) Investment by new firms responds to Tobin's Q much more elastically than does investment by incumbent firms. We argue that this is because investment of new firms crowds out investment by incumbents more when Q is high than when Q is low. Paradoxically, the investment of incumbents is highly elastic and, for that very reason, easy to crowd out with little effect on stock prices.

1 Introduction

Investment in new firms appears to be significantly more elastic with respect to Tobin's Q than investment of established firms. We show this with aggregate measures. We argue that this is because investment in new firms crowds out investment by incumbents more when Q is high than when Q is low. Paradoxically, the investment of incumbents is arguably highly elastic and, for that very reason, easy to crowd out with little consequence for stock prices.

Most economic models imply that investment should respond positively to movements in Tobin's Q. Yet, measured investment of firms shows little response to movements in measured Tobin's Q. So little, in fact, that one needs puzzlingly high capital-adjustment costs to explain the pattern. The puzzle is there in aggregate data and at the firm level.

No such puzzle exists for investment in new firms, however. Venture capitalists invest almost exclusively in young start-up firms. Venture investment as we know it today did not really get off the ground until the 1980s. Figure 1 shows that such

*We thank Robert Lucas for comments and Hakon Tretvoll and Viktor Tsyrennikov for research assistance.

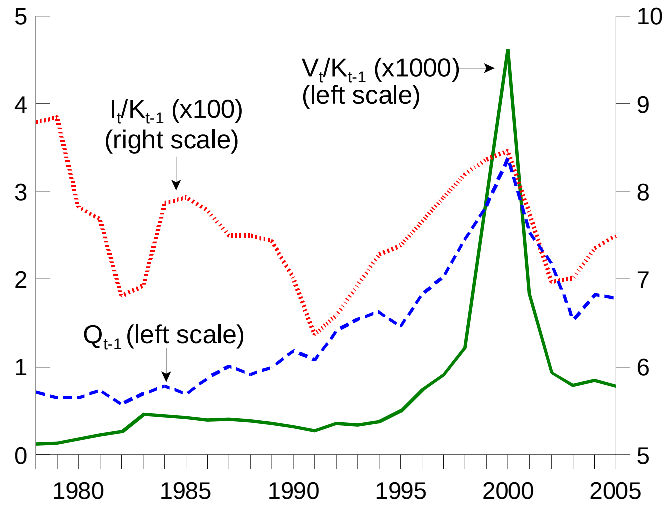


Figure 1: Venture investment, aggregate investment, and Q 1978-2005

investment responds elastically to Q, quite in contrast to aggregate investment which bears little relation to Q. The correlations with Q are 0.844 and 0.186, respectively.[1]

The same is true for IPOs; while IPOs are an imperfect and delayed measure of investment in new firms, they are better than any other century-long time series that is available.[23] They are shown along with Q and aggregate investment in Figure 2.[4]

[1]In Figure 1, data on venture capital investment are from the "Venture Xpert Database" of Thompson Venture Economics, Inc, and represent flows over each calendar year from 1978 through 2005. K_t is measured as the year-end stock of private fixed assets from the detailed fixed assets tables of the Bureau of Economic Analysis (2006, Table 6.1, line 1). I_t is gross private fixed investment from the National Income and Product Accounts. For Tobin's Q, we use fourth quarter observations underlying Hall (2001) for 1978-1999, and ratio splice estimates from Abel (????) to Hall's series for 1999 to 2005.

[2]The incorporations data and establishments data are dominated by dry cleaners, corner stores and such, and therefore not much to do with the model.

[3]Of course it is reasonable that investment would not respond to Q in the region where $Q < 1$. If adverse shocks cause the value of capital in place to fall, and if firms cannot reduce the stock of their capital, the value of their capital can fall below its replacement cost and nothing will happen to investment. In a representative firm model, however, this hypothesis (Sargent 1980) faces the problem that aggregate investment is always positive. When firms can differ, the Qs of some firms would fall below one, while for others – and hence for the economy at large – investment would remain positive. One problem with this is that the capital-weighted fraction of plants with zero investment never rises above 6 percent (Kashyap and Gourio 2007, Figure 1), and so there seems little chance that such a model can ever realistically deliver an aggregate Q below unity.

[4]In Figure 2, K_t is the end-of-year stock of private fixed assets from the BEA (2006, Table 6.1,

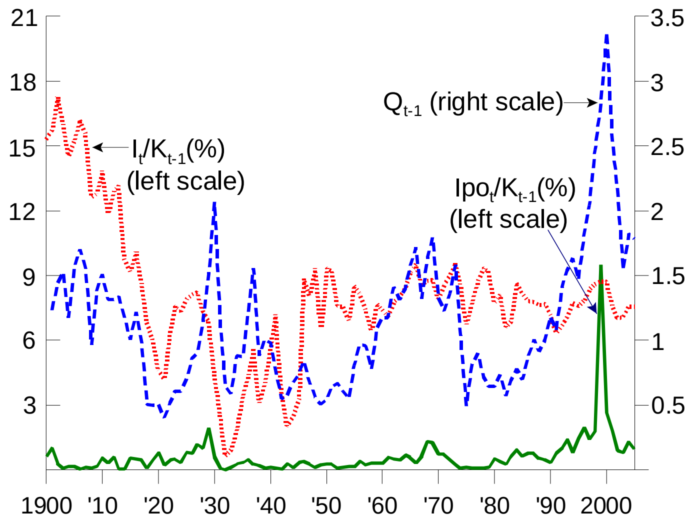


Figure 2: IPOs, aggregate investment, and Q : 1900-2005

Over the past 115 years, and respective correlations are 0.574 and 0.305, while for 1954-2005 the correlations are 0.635 and 0.246.

We shall follow the vintage-capital tradition and stress heterogeneous investment technologies. The heterogeneity will be over different vintages of firms.[5] Instead of a

line 1) for 1925 through 2005. For 1900-1924, we begin with annual estimates from Goldsmith (1955, Vol. 3, Table W-1, col. 2, pp. 14-15) that include reproducible, tangible assets (i.e., structures, equipment, and inventories), and then subtract government structures (col. 3), public inventories (col. 17), and monetary gold and silver (col. 18). We ratio-splice the result to the BEA series. IPOs are measured as the total year-end market value of the common stock of all firms that enter the database developed by the University of Chicago's Center for Research in Securities Prices (CRSP) in each year from 1925 through 2005, excluding American Depository Receipts. The CRSP files include only listings from the New York Stock Exchange (NYSE) from 1925 until 1961, with American Stock Exchange and NASDAQ firms joining in 1962 and 1972 respectively. This generates large entry rates in 1962 and 1972 that for the most part do not reflect initial public offerings. Because of this, we compute the average of the entry rates in 1961 and 1963 and in 1971 and 1973, and assign these averages to the years 1962 and 1972 respectively. For 1900-1924 we obtain market values of firms that list for the first time on the NYSE using the pre-CRSP database of stock prices, par values, and book capitalizations developed in Jovanovic and Rousseau (2001, see footnote 1, p. 1). We continue to use Hall's (2001) fourth quarter data to bring Q_t back to 1950, and then ratio splice the "equity Q" measure underlying Wright (2004) to take the series back to 1900. Note that Hall's measure of Q_t exceeds Wright's by factor of more than 1.5 in 1950, when the splice occurs, producing Qs before 1950 that are considerably higher than Wright's original estimates.

[5] Jovanovic and Rousseau (2001) show that old firms and firms with old capital trade at a discount, especially in sectors where technological progress is rapid. Thus more efficient new capital devalues

full-blown vintage-capital model, however, we model an economy in which capital is homogeneous, but in which entrants and incumbent firms have different investment technologies. Entrants will face convex capital-adjustment costs in the tradition of Lucas (1967) and Lucas and Prescott (1971), whereas incumbents will have constant but random costs of investment. The treatment will be a stochastic version of the 'spin-out' model Prescott and Boyd (1987).

Our model will, however, reverse a line of causation usually present in the vintage-capital model. As in the vintage capital model, the value of old capital is determined by the state of the investment technology – the technology of the latest vintage determines the market value of capital of earlier vintages. In our model, by contrast, shocks to the investment technology of incumbent firms determine their own Tobin's Q and the value of creating new firms. This small change in the vintage-capital model is important for explaining the pattern shown by Figures 1 and 2.

The model shares the features of the vintage-capital model in that high stock prices (i.e., a high value of capital in place) are a signal of an unfavorable shock to the technology of creating new capital or, rather, capital of the latest vintage.

Several papers endogenize the number of firms over the business cycle, but none treats the extensive and intensive margin in the way that we do. Campbell (1998) and Bilbiie, Ghironi and Melitz (2006) endogenize the number of firms in business-cycle models but both papers treat the capital of incumbents as fixed at its entering value.

2 Model

Aggregate output is zk. There is one capital good, k, but two ways to augment it: via investment of incumbents, X, and via investment of entrants, Y, so that capital evolves as

$$k' = (1-\delta)\,k + X + Y. \tag{1}$$

The aggregate resource constraint expresses the consumption of the representative agent as

$$C = zk - qX - h\left(\frac{Y}{k}\right)k \tag{2}$$

The RHS of (2) is linear homogeneous in (k, X, Y).

On the RHS of (2), the two forms of investment cost are subtracted from output. The first cost, qX, is interpreted as the investment costs borne by incumbent firms which face a constant cost, q, per unit of capital created. We assume that q is random. In our model, q reflects the efficiency of entrants relative to incumbents. In the spirit of vintage-capital models, when q is high, the cost of entering capital is low relative

old capital. Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001) have used this logic to argue that technological progress caused Q to fall below 1 in the mid 70s and remain there for 10 years.

to the cost of expanding the capital of incumbents. The parameter q will be regarded as a shock that the BLS does not observe. Greenwood, Hercowitz and Krusell (1997, 'GHK') argue in a two-sector model that $1/q$ is the productivity of the capital-goods sector and is directly related to the measured price of capital. Here, however, q is a shock *relative* to any price of capital that anyone can measure. The measured cost of capital is later asserted to be unity. Moreover, and in contrast to GHK, k is a capital aggregate, an amalgam of physical and human capital, and so q includes (maybe predominantly so) shocks to the training function.

The other cost, $h\left(\frac{Y}{k}\right)k$, represents the costs of creating entering capital. We follow Prescott and Boyd (1987) and assume that every unit of entering capital is 'spun out' by incumbents. In return for providing the investment needed to create that spinout, the incumbent pays commensurately lower earnings to the workers that will end up managing the spinout. This arrangement can work because, as Becker (1993) explains, general training should be financed by the worker.[6]

Two comments now on how we shall match the data to the theoretical concepts of cost in (2):

1. *The asymmetry in the cost of new and old capital.*—The cost of an incumbent firm's capital is qX, and that of entering firms is $h\left(\frac{Y}{k}\right)k$. We think of q as the unit cost of routine investment, and of h as the cost of doing new things. New ideas vary in quality and the good ones yield a higher return. The best ideas are exploited first, hence h' is upward sloping. The microfoundations of spinouts involve asymmetric information. Klepper and Thompson (2006) argue that spinouts occur because people disagree on management decisions – people leave firms to develop their ideas on their own because coworkers would otherwise implement the idea suboptimally. Chatterjee and Rossi-Hansberg (2007) is about adverse selection – the best ideas leave the firm. Some evidence for this is in Prusa and Schmitz (1993).

2. *No externalities.*—We shall be assuming a representative firm that treats the cost $h\left(\frac{Y}{k}\right)k$ as internal. If Y stands for all new capital in new firms, this means that new firms can be formed only by paying the cost $h\left(\frac{Y}{k}\right)k$. Each new firm, therefore, is a product of activity done in incumbent firms. The closest-fitting activity is corporate venturing or spin outs started by ex-employees of incumbent firms. In reality, new capital forms in several ways. There is investment by incumbent firms. Then there is capital formation by spinouts. Finally, there are de-novo startups by people with no relevant labor-market experience. Only the first two categories are covered by the RHS of (2), but perhaps the third does not matter quantitatively. This is analytically extremely convenient, for

[6] Other models in this spirit are Chari and Hopenhayn (1991), Franco and Filson (forthcoming) and Chatterjee and Rossi-Hansberg (2007).

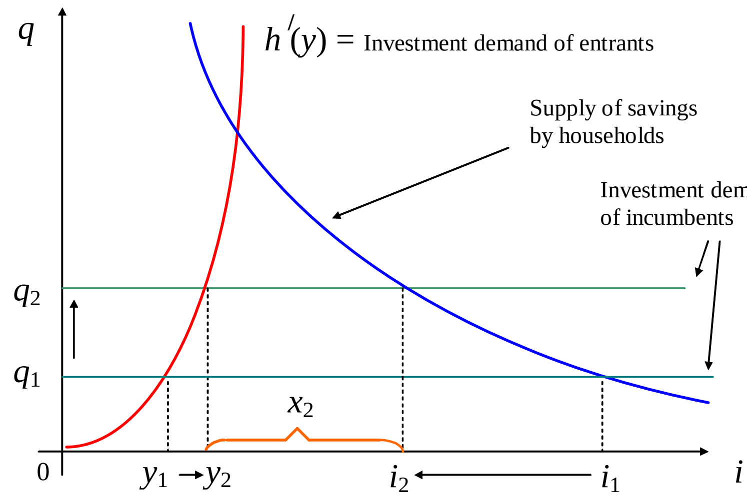


Figure 3: THE DETERMINATION OF y VIA (5)

it implies that equilibrium can be calculated via the planner's optimum.[7]

The determination of investment.—We shall now show that the ex-dividend price of capital will equal q. The investment rate of entering capital is fully determined by, and increasing in q, as shown in Figure 3. Incumbent investment will take up the slack between desired total investment and the investment of entrants. There will be a second shock, z, and the supply of savings and, hence, the residual incumbent investment will depend on both q and z. Figure 3 illustrates the effect of a rise in q when z is held constant. The 'interiority' requirement that $x > 0$ implies that we can determine y from the intersection of the entrants' investment-demand curve $h'(y)$ with q. As q rises while z stays fixed, two things happen: First, savings declines and with it total investment i must fall from i_1 to i_2. Second, the supply of entrants rises from y_1 to y_2, thereby crowding out an even larger amount of incumbent investment. A reading of Gompers (2004) would suggest a high degree of substitutability between a corporation's in-house investment in physical and R&D capital and its venturing investments.[8]

[7] One could, instead, treat a part of the costs as external, such as

$$kh\left(\frac{Y}{k}\right)G\left(\frac{\mathbf{Y}}{\mathbf{k}}\right),$$

with k the component internal to the firm, and $(\mathbf{Y}, \mathbf{k})$ as the external part. The firm's problem would remain linearly homogeneous, and equilibrium would remain qualitatively the same. Equilibrium growth, however, would generally no longer be efficient.

[8] On the other hand, only about ten percent of venture investments are made by corporations – see Chart 4 of Gompers (2004).

The economy has no external effects or monopoly power and equilibrium can be represented by a planner's problem. This problem has a traditional one-sector representation with a single cost of adjusting the economy-wide capital stock. That adjustment cost function will be a reduced form, representing the outcome of a static allocation problem, namely one of minimizing the cost of providing a certain amount of new capital conditional on the realization of q alone.

2.1 The Planner's problem

Preferences are $E_0\left\{\sum_0^\infty \beta^t U\left(C_t\right)\right\}$. Let $s \equiv (q, z)$ be stochastic with transition function $F\left(s', s\right)$. The state of the economy is (s, k), but since returns are constant and preferences homothetic, k will not affect prices or investment rates. The planner's problem is to maximize the representative agent's expected utility by choosing the two kinds of investments X and Y. The planner has no other technology. The Bellman equation is

$$V(s, k)=\max_{X \geq 0, Y \geq 0}\left\{U\left(zk-qX-h\left(\frac{Y}{k}\right) k\right)+\beta \int V\left(s',(1-\delta) k+X+Y\right) dF\left(s', s\right)\right\}. \tag{3}$$

Let $y=Y/k$ and $x=X/k$. The FOCs are[9]

$$-qU'+\beta \int V_k dF=0, \tag{4}$$

and

$$-h'(y) U'+\beta \int V_k dF=0.$$

We shall assume throughout that both FOCs hold with equality. To ensure this, we assume that $h(0)=0$, which rules out the value $Y=0$, and that (ii) $h'(y)>q_{\max}$ at a value of y too low to satisfy the demand for saving in any state s, which rules out $X=0$.[10] Combining the two FOCs leads to

$$h'(y)=q. \tag{5}$$

as illustrated in Figure 3. Therefore $y(q) \equiv \left(h'\right)^{-1}(q)$ is increasing in q.

If y always satisfies (5), we can write (3) as[11]

$$V(s, k)=\max_{k' \geq(1-\delta) k}\left\{U\left(zk-q\left(k'-(1-\delta) k-y(q) k\right)-h(y(q)) k\right)+\beta \int V\left(s', k'\right) dF\left(s', s\right)\right\}$$

[9] Differentiability of V in k will be shown below.

[10] This requirement will be considered explicitly when we solve for the growth rate in the deterministic case in Section 2.2.

[11] Evidently, the model has a standard one-sector representation. Let $I=X+Y$ and let

$$f(i, q) \equiv \min_x\{qx+h(y)\} \qquad \text{subject to } x+y=i.$$

whence

$$\begin{aligned} V_k &= U'(C)(z+q(1-\delta+y(q)) - h(y(q))) = U'(C_t)(z_t + q_t(1-\delta+y_t) - h(y_t)) \\ &= U'(C_t)\left(\frac{D_t}{k_t} + q_t\left(\frac{k_{t+1}}{k_t}\right)\right) \end{aligned}$$

because $= 1-\delta+y_t = \frac{k_{t+1}}{k_t} - x_t$. Therefore the price of a unit of capital satisfies[12]

$$q_t = \beta \int \frac{U'(C_{t+1})}{U'(C_t)} \left(\frac{D_{t+1} + q_{t+1}k_{t+2}}{k_{t+1}}\right) dF(s_{t+1}, s_t). \tag{6}$$

This is the discounted expected value of the firm's earnings and in the standard decentralization as done by Lucas (1978) would be the price of capital that shareholders pay. The replacement cost of capital is unity, and therefore q is also what is known as Tobin's Q. We should think of q_t as the price of capital at the *end* of the period, since a purchase at date t at that price does not entitle the holder to period-t dividends.

Let preferences be

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

in which case the value function must satisfy

$$V(s,k) = v(s)\, k^{1-\sigma}, \tag{7}$$

where

$$v(s) = \max_{x,y} \left\{ \frac{(z - qx - h[y])}{1-\sigma}^{1-\sigma} + (1-\delta+x+y)^{1-\sigma} v^*(s) \right\} \tag{8}$$

and

$$v^*(s) = \beta \int v(s')\, dF(s', s).$$

If the processes z and q are positively persistent and if they are mutually independent, v and v^* are strictly increasing in z and strictly decreasing in q.

The investment policies.—By (5) we know that y depends on q alone and is increasing in q. Regarding x, we have

---

Then the economy is equivalent to one in which, instead of (1) and (2), we have

$$C = zk - f\left(\frac{I}{k}, q\right) k \qquad \text{and} \qquad k' = (1-\delta)k + I.$$

[12] If x_{t+1} and y_{t+1} both were to equal zero, then we would get the more intuitive expression

$$q_t = \beta \int \frac{U'(C_{t+1})}{U'(C_t)} (z_{t+1} + (1-\delta) q_{t+1}).$$

Proposition 1 *x is (i) strictly increasing in z, and (ii) strictly decreasing in q.*

Proof. (i) The RHS of (9) is strictly increasing in z directly and increasing through v^*, and it is strictly decreasing in x. The LHS of (9) does not depend on z. Then the implicit function theorem yields the first claim. (ii) From (7),

$$V_k(s,k) = (1-\sigma)\, v(s)\, k^{-\sigma},$$

and then (4) reads

$$\begin{aligned} q &= \frac{1-\sigma}{U'}\left([1-\delta+x+y]\,k\right)^{-\sigma}\beta\int v(s')\,dF(s',s) \\ &= (1-\sigma)\left(\frac{z-qx-h(y)}{1-\delta+x+y}\right)^{\sigma} v^*(s) \\ &= (1-\sigma)\left(\frac{z-qx-h\left((h')^{-1}(q)\right)}{1-\delta+x+(h')^{-1}(q)}\right)^{\sigma} v^*(s), \qquad (9) \end{aligned}$$

which allows us to solve for x as a function of $v^*(s)$. Since $(h')^{-1}(q)$ is increasing in q, the term $(\cdot)^{\sigma}$ is strictly decreasing in q, and so is v^*. Therefore the RHS of (9) is strictly decreasing in q. As we mentioned under (i), the RHS of (9) is strictly decreasing in x. Since the LHS of (9) is increasing in q, the implicit function theorem then delivers the second claim. ■

2.2 Decentralization 1: A corporate venturing economy

We can think of $h\left(\frac{Y}{k}\right)k$ as the cost of discovering new investment opportunities adding up to Y units of tomorrow's capital. Experience helps in such discovery and therefore k lowers its cost. This fits the activity of venture investment by corporations. These corporations retain ownership of the dividends that these investments will provide in the future. We discuss later the empirical significance of such activities. The component qX represents the costs of reinvesting in routine activities.

We shall assume that the firm maximizes the value of its current shareholders, measured in units of current consumption. As Lucas and Prescott (1971) explain, the firm takes as given the next-period value of its capital, its ex-dividend value today being q per unit of capital created. The firm's maximization problem is then

$$w = \max_{x,y}\left\{z - qx - h(y) + (1-\delta+x+y)\,q\right\}. \qquad (10)$$

This means that the firm's y must solve (5), but x must be obtained from the household savings decision and the identity that savings = investment = $x+y$. The linear technology for creating k does not yield any rents because the unit price of capital adjusts to equal the average and marginal cost of creating it. Therefore (10) reduces to

$$w = z + q(1-\delta) + \max_{y}\{qy - h(y)\}.$$

We shall only sketch this (MORE NEEDED HERE) since it straightforwardly adapts Lucas (1978). The household maximizes discounted expected utility subject the constraint

$$qn' + c = (z - x + q)\,n$$

where n is the number of shares of the representative firm that the household owns and where $c = C/k$. The household's FOC and that of the firm then lead to (6).

2.3 Decentralization 2: A spin-out economy

We can, instead, think of $h\left(\frac{Y}{k}\right)k$ as the cost (either direct or in the form of foregone output) of providing the firm's workers with the training needed to discover new investment opportunities, but where the implementation of these opportunities is done by the workers after they leave the firm. The parent corporation now does not own the dividends that these investments will provide. Rather, it will charge for the training that it provides by paying lower wages. This decentralization will be essentially that of Prescott and Boyd (1987, 'PB'), and in the spirit of the analyses of general human capital in Becker (1993). People live for two periods and have preferences $E\{U(c_Y) + \beta U(c_O)\}$ where c_Y is consumption when young and c_O consumption when old. The young inherit all the capital that the firm creates, but a certain fraction of them, Y_t/k_{t+1}, start new firms. The rest stay and continue to operate the existing entity.

2.3.1 The version with size of firm exogenous

Output can be produced only if an old worker (the 'manager') and a young 'worker' are present. Population is constant and each firm is composed of an equal number of old and young workers. Let k be the human capital of the managers. Let $k' = k + I$ be the total human capital given to each young worker. Net output per unit of k is $z - f(i, q)$ where, as discussed in footnote 10, the firm's investment-cost minimization problem is

$$f(i,q) \equiv \min_{x,y}\{qx + h(y)\} \text{ subject to } x + y = i. \tag{11}$$

That is, the firm provides the capital as cheaply as possible so that y still solves (5). We then say that a fraction

$$\frac{y(q)}{1-\delta+i(s)}$$

of the workers start new companies, while the rest stay with the same company.

Let k or, rather, the firms be the only asset. A firm is owned by its manager(s). No other assets exist in this economy but because returns are constant there is no borrowing and lending among firms in the equilibrium that we shall describe.

Because the young care about lifetime expected utility, the manager will choose the wage-training package that provides his one worker the equilibrium utility as efficiently as possible. Suppose that the manager consumes $kp(s)$. We take the function $p(s)$ as given for now, and we assume it to be independent of k. The lifetime utility of the worker, ϕ, is

$$\phi(s,k,p(.)) = \max_i \left\{ U(k(z - f(i,q) - p(s))) + \beta \int U(k(1-\delta+i)p(s'))\, dF(s',s) \right\}.$$

Since $\frac{\partial f}{\partial i} = q$, this gives rise to the investment policy $i(s)$ satisfying

$$q = \frac{1}{U'(C_Y)} \beta \int p(s') U'(C_O)\, dF(s',s) \tag{12}$$

With $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$, (12) reads

$$q = \frac{(1-\delta+i)^{-\sigma}}{(z - f(i,q) - p(s))^{-\sigma}} \beta \int [p(s')]^{1-\sigma}\, dF(s',s), \tag{13}$$

and it is therefore indeed feasible, as we assumed, that $p(s)$ and i should be independent of k. Eq. (13) Therefore it is as if the worker buys the firm from the manager at a fee $kp(s)$ and maximizes his utility with the expectation that he will receive $k'p(s')$ next period.

There appear to be many other equilibria in this OLG economy, each corresponding to a different weight of the young and old in the sharing of output. This is a point that Cozzi (1998) has already made in a similar setting. An equilibrium to this situation is the one that we already displayed, namely, one in which everyone consumes a constant fraction of output in each period. In that case, (12) and (4)

$$-qU' + \beta \int V_k dF = 0, \tag{14}$$

are the same, because when everyone consumes the same amount, $\int V_k dF = E_t U'(C_{t+1})$, and therefore $p_t = q_t$.

Example.—Let $\sigma = 1$. Then (13) reads $q = \beta \frac{z - f(i,q) - p(s)}{1-\delta+i}$, whence we get

$$i + \frac{\beta}{q} f(i,q) = \frac{\beta}{q}(z - p(s)) - (1-\delta). \tag{15}$$

Since $\frac{\partial f}{\partial i} = q$, the LHS has a derivative w.r.t. i equal to $1+\beta$ as long as the solutions for x and y are interior. Therefore, $i = \mu - \frac{\beta}{q(1+\beta)} p(s)$,where $\mu > 0$ is a constant.

Therefore, equilibria with a larger share of the old imply less investment and less growth. On reflection we should have expected this, because (i) If this were a savings problem, a change in the return on capital would have exactly offsetting income and substitution effects, and (ii) A larger p implies a subtraction from the output left over for the consumption and investment of the young. Therefore there is a second negative income effect that causes investment to fall because consumption is a normal good.

2.3.2 The version with size of firm endogenous

Endogenizing firm-size.—We follow PB and reach a unique solution by introducing a firm-size margin. To narrow down the equilibrium, PB append a decision about firm size as follows: Let n be the firm's employment and let the firm's output be

$$\text{output} = kQ(n)$$

where k is the quality of the manager, as in Lucas (1978a). Let k, X, and Y be capital and investments per worker. Let the firm's investment costs depend only on the totals accumulated[13], nX and nY, so that these costs are $qnX - h\left(n\frac{Y}{k}\right)k$. The firm's revenue per unit of k then is

$$zQ(n) - qx - h(ny),$$

where Q is increasing and strictly concave. For $n \neq 1$, the firm's subproblem (11) becomes

$$f(i,q,n) \equiv \min_y \{qn(i-y) + h(ny)\} \tag{16}$$

and when evaluated at $n = 1$, the FOC is still (5).

Since the firm's output is shared among n workers, the worker-participation constraint now reads

$$\max_i \left\{ U\left(\frac{k}{n}(zQ(n) - f(i,q,n) - p(s))\right) + \beta \int U(k(1-\delta+i)p(s'))\,dF(s',s) \right\}.$$

Normalizing $Q(1) = 1$, when evaluated at $n = 1$, the FOC w.r.t. i is still (12). Finally, the FOC w.r.t. the new choice variable, n, is

$$-(z - f(i,q,1) - p(s)) + zQ'(1) - q(x+y) = 0,$$

in light of (5). Then since $q(x+y) - f(i,q,1) = qy - h(y)$,

$$p(s) = z(1 - Q'(1)) + qy - h(y) > 0. \tag{17}$$

[13] This is different from eq. (1) of PB

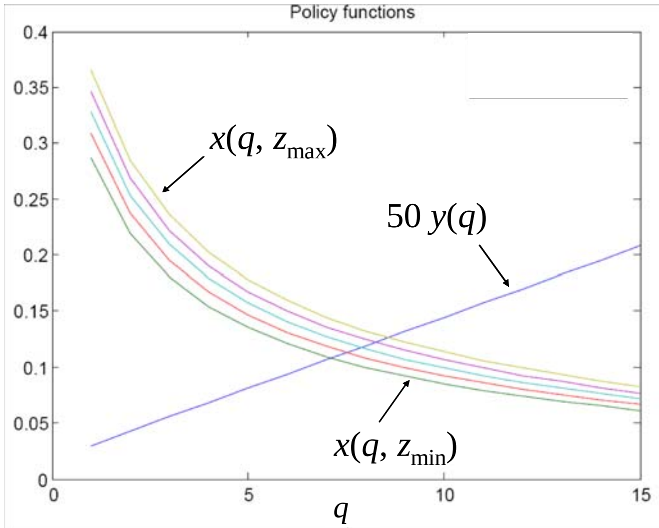


Figure 4: Optimal investment policies

The inequality follows because (i) $1-Q'$ is the average minus the marginal product of a unit measure of workers and (ii) Since $h' = q$, $qy - h$ is the marginal minus average cost.

Example: Once again, let $\sigma = 1$, $Q(n) = n^{\alpha}$, and $h(y) = \gamma y^2/2$. Then

$$p(s) = (1-\alpha) z + \frac{q^2}{2\gamma}$$

$Q'(1) = \alpha$, and $h(y) = \gamma y^2/2$. In that case Since (15) which is still valid, we substitute into it for $p(s)$ from (17) to get

$$i + \frac{\beta}{q} f(i, q) = \alpha \frac{\beta}{q} z - (1-\delta) - \beta \frac{q}{2\gamma}$$

which has exactly one solution for i whenever the parameters are chosen so that RHS is always positive.

3 Simulation and data fitting

Figure 4 shows the optimal investment policies for the parameters $\beta = 0.95$, $\sigma = 6$, $\delta = 0.1$, and $\gamma = 771$, and for a discretized version of the (q, z) process. The parameters of the (q, z) precesses were chosen to be AR(1) and fitted via the Tauchen-Hussey procedure. We let z take on 5 values and q take on 15 values. The statistical properties of the discretized processes were then presented to the Planner who chose the optimal policies. These are plotted in Figure 4.

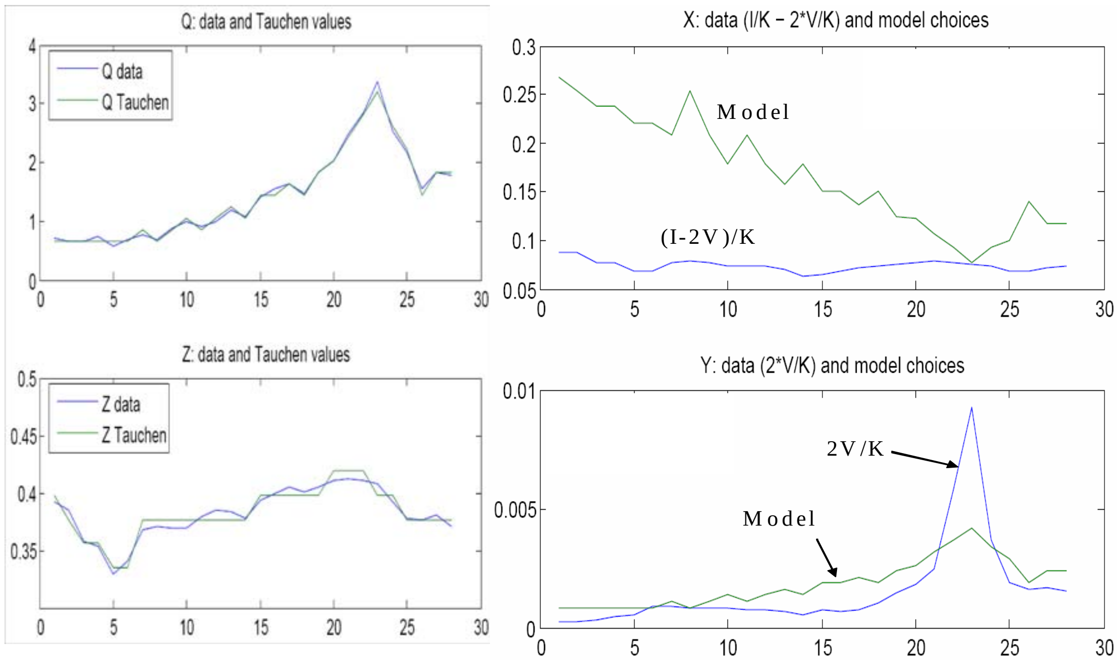


Figure 5: Simulation and fit

Figure 5 presents the fit of the model. The two left panels of Figure 5 show that the discretized processes track the actual values extremely well. The bottom left panel of Figure 5 shows that the model fits venture investment pretty well. The top right panel shows, however, that the model vastly exaggerates the negative relation between Q; the data seem to show an incumbent investment that is flat. This mismatch of the model and data probably arises because in the model a rise in Q reflects only a rise in the cost of capital whereas in reality it sometimes may indicate a rise in the marginal product of capital, i.e., z. Because of the infinite elasticity of supply of incumbent capital at q, the model shuts off the channel that may produce the latter effect.

3.1 Regressions with data in Figures 1 and 2

Measurement.—To take our theory to the data, we need measures of the state variables q and z, and of x and y. In light of (6), we measure q by Tobin's Q. Since output is zk, we measure z by the ratio of private output over the course of a given year to private capital at the start of that year. This measure is not accurate for the period from the start of U.S. involvement in World War II until several years after the war because, as Gordon (1960) explains, capital used by private firms was sometimes classified as Government capital, and this would $\hat{z}$ to be biased upwards. For this reason we exclude the 1941-1952 period from our regression analysis.

We entertain two measures for y_t. The first is total investment of venture capital

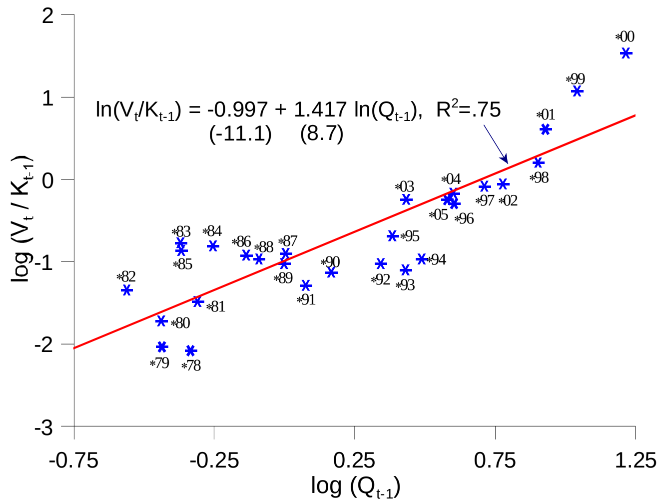


Figure 6: THE RELATION BETWEEN VENTURE INVESTMENT AND Q, 1978-2005

funds. It covers a significant fraction of startup investment, and very little else. Occasionally, venture capital (VC) funds are involved in taking mature companies public, but this is rare and the investment involved is small. Thus, while our series for VC-investment series excludes new firms that are not backed by venture capital, it does not include anything that we would call incumbent investment. Figure 6 shows the strong and positive univariate relation between the log of V_t, defined as the ratio of total VC investment per \$1000 of beginning-of-year capital, and the log of Q at the start of each year from 1978 to 2005.

Our second measure for y_t is the total year-end market value of firms that had an IPO during year t, which we denote by IPO capitalization. Though with a lag, this measure will include not only VC investments but also corporate venturing investments (Gompers 2004).[14] It is remarkable how similar the corporate-venturing portfolio firms are to the independent VC-backed portfolio companies. They are concentrated only slightly in investment rounds that come later than the typical rounds of independent-VC-backed investments, but are otherwise quite similar in terms of the size or the investments and industries covered. Corporate-backed ventures are more likely to reach IPO (Gompers 1994, Table 6), which runs counter to popular perception that corporations invest in young startups in order to gobble them up once they reach a viable stage. Recalling that q_t is the end-of period price of capital, the date-t value of entering-firms' capital relative to the value of *beginning*-of-period

[14]During the late 1960s and early 1970s, more than 25 percent of the Fortune 500 firms set up divisions that emulated venture capitalists.

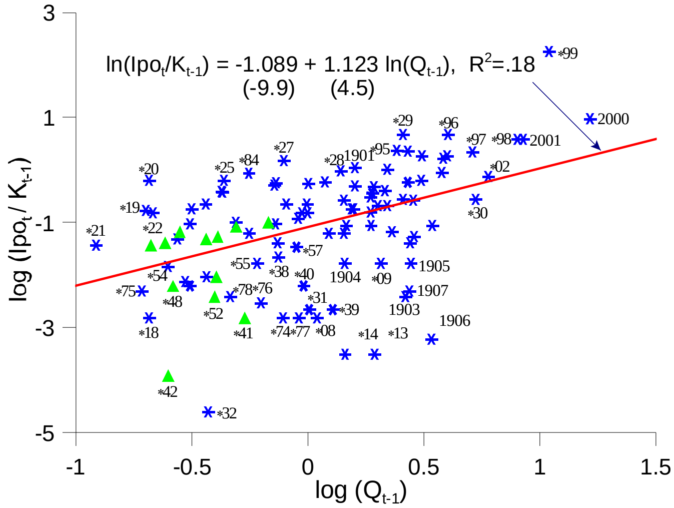


Figure 7: THE RELATION BETWEEN IPOS VALUES AND Q

capital is

$$\varepsilon_t^* = \frac{q_t y_t k_t}{q_{t-1} k_t} = \frac{q_t}{q_{t-1}} h'^{-1}(q_t) \tag{18}$$

(from (5)). Then the model says that z should not enter this equation once q_t and q_{t+1} are held fixed. If adjustment costs are quadratic in y: $h(y) = \gamma y^2/2$, then $y_t = q_t/\gamma$, and so (18) reads

$$\varepsilon_t^* = \frac{q_t^2}{\gamma q_{t-1}}. \tag{19}$$

or, taking logs, $\ln \varepsilon_t^* = -\ln \gamma + 2 \ln q_t - q_{t-1}$. We also note that z_t does not enter this regression. We measure $q_{t+1} y_t k_t$ is the end-of-t value of all the firms that listed during year t, and $q_t k_t$ is the value of firms listed at the beginning of year t. Figure 7 once again shows a strong and positive univariate relation between this second measure of y and Q.[15]

We also entertain two measures for x. The first is private investment, I_t, deflated by the private capital stock at the start of the year, K_{t-1}. This would be the right measure if all entering investment was in the form of foregone output. But venture investment, for one, is measured investment. The second measure subtracts a measure of Y from I to arrive at an estimate of X that we denote by $\hat{X}_t$. For the period 1978-2005 we define $\hat{X}_t = I_t - 2V_t$ because roughly half of the firms that have IPOs in

[15] The years from 1941 to 1953 appear as light triangles in Figures 6 and 8, and are not included in fitting the regression relationships shown.

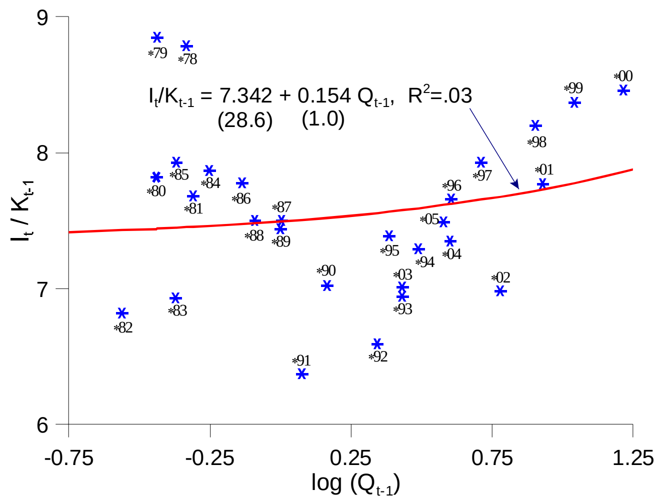


Figure 8: THE RELATION BETWEEN AGGREGATE INVESTMENT AND Q FOR THE PERIOD 1978-2005

the United States are Venture-backed. Since far fewer non-VC-backed firms ever go public, investment in these companies is probably higher than that in VC-backed firms; i.e., $\hat{X}_t$ is probably larger than X_t. The regressions in Table 1 use measures that exclude R&D spending. Figure 8 shows the weak cross-section relation between I_t/K_{t-1} and the log of Q_{t-1} for the 1978-2005 period (i.e., the period covered by Figure 6). The relation is not any stronger for the century as a whole, as Figure 9 shows.

Functional form.—The functional form for adjustment costs will be assumed to be quadratic: $h(y) = \frac{\gamma}{2}y^2$, in which case $y = \frac{q}{\gamma}$. Although we do not have x in closed form as a function of q, we shall assume that it, too, is adequately represented by a linear function of q.

Proposition 1 tells us that the two investment policies, x and y, differ qualitatively in their dependence on q and z. The investment of entrants, y, should depend positively on q, and not at all on z. Investment of incumbents, x, should be decreasing in q and increasing in z. Table 1 reports the regressions that test these implications.

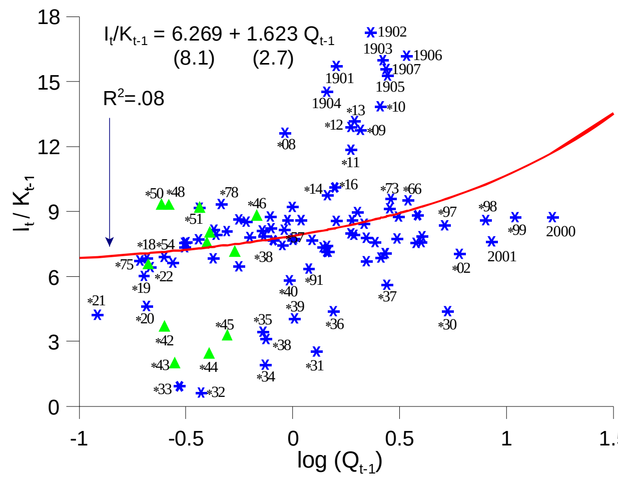


Figure 9: The relation between aggregate investment and Q for the period 1900-2005

Table 1
Entering Capital and Aggregate Investment Regressions

	1978-2005				1901-2005			
	$\ln V_t/K_{t-1}$		$\widehat{X}_t/K_{t-1}$	I_t/K_{t-1}	$\ln Ipo_t/K_{t-1}$		$\widehat{X}_t/K_{t-1}$	I_t/K_{t-1}
const.	-0.997	-6.667	2.315	3.040	-1.089	-9.417	-10.073	-10.656
	(-11.07)	(-3.54)	(0.92)	(1.11)	(-9.90)	(-2.40)	(-3.23)	(-3.87)
Q_{t-1}	1.417	1.879	-0.344	-0.082	1.123	0.911	-0.650	0.460
	(8.72)	(8.98)	(-1.74)	(-0.38)	(4.53)	(3.46)	(-1.09)	(0.87)
z_t		-5.780	0.146	0.121		2.288	0.477	0.474
		(-3.01)	(2.08)	(1.58)		(2.12)	(5.61)	(6.33)
R^2	.75	.81	.15	.12	.18	.22	.26	.36
N	28	28	28	28	93	93	93	93

Note: T-statistics in parentheses. All variables are scaled as in Figures 1 and 2. The regressions for 1901-2005 exclude the years 1941-53.

3.1.1 Relation to the micro evidence

Our measures of "entering investment" do not seem to show the same pattern as the "Class-1" (i.e., low payout) sample of Fazzari, Hubbard and Petersen (1988) or the "immature firms" sample of Chirinko and Schaller (1995). These studies work entirely with publicly-owned firms, and the authors chose their subsamples in order to identify firms that are likely to be liquidity-constrained. These authors found a smaller Q-elasticity of investment in this sample than in the samples of high payout and mature firms. It is clear that our data on entering investment cover firms that are not so liquidity constrained. That is certainly so for the VC-backed investment data. A VC-backed firm may feel that it is liquidity constrained, but the VC funds backing it usually are not, because the investment "overhang" (i.e., the amount by which monies committed to the VC fund exceed those actually used to fund their investments) typically exceeds an entire year's worth of investment. Roughly speaking, then, VC-backed firms have indirect and immediate access to about one-year's worth of investment. Indirectly then, one may be tempted to infer liquidity constraints, but this would be unwarranted. VC-backed firms are usually much more focused on state-of-the-art technologies than other small firms, and much more likely to eventually go public and for that reason if for no other, much more likely to be sensitive to variation in Tobin's Q.

The volume of IPOs, on the other hand, is a transition rate from one financing status to another. It arguably measures the rate at which firms acquire easier financing, since one motive for IPOs, it is said, is to gain access to liquidity. It is apparent that there is a "timing" aspect to the IPO decision; The owners of the firm will want to exercise the option to have an IPO when the market values are high and the owners can get the most for their shares. Rather than a measure of real investment, one can argue that what we measure is simply the exercise of a financial option. On the other hand, evidence also shows (Chemmanur, He and Nanda 2005, Figure 3) that a firm's investment rises by the non-trivial factor of 1.4 around the time of IPO ($\pm$ 2 years), which means that IPOs also measure a rise in investment. But such investment is perhaps less "entering investment" than VC-backed investment is.

3.1.2 Vintage capital and the homogeneity of k

UNFINISHED SECTION The model assumes that once created, capital is homogeneous. Only the cost of creating it differs among agents. After entry, the capital of different vintages does not change relative to that of other vintages. The loss of market share to entrants is therefore shared by incumbents of all vintages equally. This subsection checks how well this assumption fits the facts

Figure 10 shows how well over time the IPO-ing firms of each decade performed relative to firms that existed before them.[16] There is some downward trend in these

[16]Figure 10 is based on end-of-year market capitalizations from the CRSP files for 1925 to 2006,

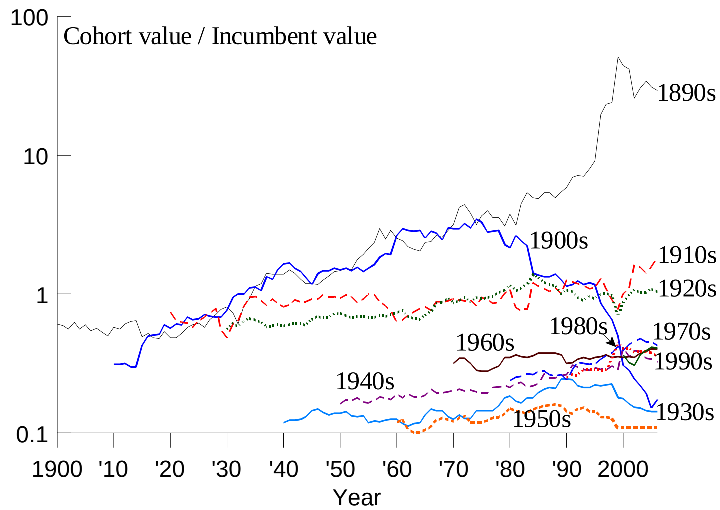


Figure 10: Shares of decadal vintages relative to earlier vintages

series because the stock market has grown over time. Thus, for example, the IPOs of the 1890s which included AT&T and General Electric, were large relative to the value of the stock market in 1890 partly because not so many firms were listed in 1890.

These measures look at surviving value, and not returns. If dividend policies differed by cohort so that if, for example, the 1890's cohort tended to pay less dividends and retain and reinvest more of its earnings, changes in value would reflect partly such a difference in reinvestment policies. An alternative measure is cohort returns. Gerdes (1999) has studied the returns patterns by stock-market cohort and found that older (but not very old) vintages have a higher return than the representative rebalanced index. Our vintage value plot in Figure 10 shows that no tendency for value of any particular decade to appreciate more than the others. The 1890s cohort does well and includes GE and AT&T and has, since the 1970s, done much better the other cohorts, especially the 1900-1910 cohort.

This is to be related to two sets of findings in the Finance literature. Fama and French (2000) say there is substantial mean reversion in earnings at the firm level. This is consistent with earnings being completely stable at the level of the cohort as our model implies. For instance, z can have a mean-reverting but firm-

and our backward extension of the CRSP files for 1890 to 1924. IPO years are recorded as those in which firms enter our database.

specific component, but the law of large numbers would remove the influence of this component on the relative valuation of cohorts. When divided by k_t, aggregate earnings, zk_t, are also mean reverting in the model because z is stationary. The second is the systematic IPO underpricing which implies that new firms are initially undervalued when they IPO, a phenomenon that inflates the stock-market returns of young capital relative to old capital.

4 Conclusion

We found that various measures of investment by new firms show that such investment responds to Tobin's Q much more elastically than does investment by incumbent firms which responds hardly at all. We argue that this is because investment of new firms crowds out investment by incumbents more when Q is high than when Q is low. Paradoxically, the investment of incumbents is highly elastic and, for that very reason, easy to crowd out with little effect on stock prices.

References

[1] Becker, Gary. *Human Capital*. 3rd Ed. 1993.

[2] Bilbiie, Florin, Fabio Ghironi and Marc Melitz. "Endogenous Entry, Product Variety, and Business Cycles." 2006.

[3] Campbell, Jeffrey. "Entry, Exit, Embodied Technology, and Business Cycles." *Review of Economic Dynamics* 1, no. 2 (April 1998): 371-408.

[4] Chari, V. V. and Hugo Hopenhayn. "Vintage Human Capital, Growth, and the Diffusion of New Technology." *Journal of Political Economy* 99, no. 6 (December 1991): 1142-65.

[5] Chatterjee, Satyajit and Esteban Rossi-Hansberg. "Spin-offs and the Market for Ideas." mimeo. 2007

[6] Chemmanur, Thomas, Shan He and Debarshi Nandy. "The Going Public Decision and the Product Market" Boston College, May 2005.

[7] Chirinko, Robert and Huntley Schaller. "Why Does Liquidity Matter for Investment Equations?" Journal of Money, Credit and Banking 27, no. 2 (May 1995): 527-48.

[8] Guido Cozzi. "Culture as a Bubble." *Journal of Political Economy* 106, no. 2 (April 1998): 376-394.

[9] Fama, Eugene F., and Kenneth R. French. “Forecasting Profitability and Earnings.” *Journal of Business* 73, no. 2 (April 2000), pp. 161 - 175.

[10] Fazzari, Hubbard and Petersen. “Financing Constraints and Corporate Investment.” Brookings Papers 1988, no. 1 (1988): 141-206.

[11] Franco, April and Darren Filson. “Spin-outs: Knowledge Diffusion through Employee Mobility.” *Rand Journal of Economics*, forthcoming.

[12] Gerdes, Geoffrey. *The Construction and Performance of Vintage Portfolios for the NYSE, 1926-1996*. Ph.D. Dissertation, Department of Economics, UCLA, 1999.

[13] Goldsmith, Raymond W., *A Study of Savings in the United States*. Princeton, NJ: Princeton University Press, 1955.

[14] Gompers Paul. “Corporations and the Financing of Innovation: The Corporate Venturing Experience.” *FRB Atlanta Economic Review* (Fourth Quarter 2002): 1-18.

[15] Gordon, Robert J. “$45 Billion of U.S. Private Investment Has Been Mislaid.” *American Economic Review* 59, no. 3. (June 1969): 221-238.

[16] Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. “Long-Run Implications of Investment-Specific Technological Change.” *American Economic Review* 87, no. 3 (June 1997): 342-62.

[17] Greenwood, Jeremy and Boyan Jovanovic. “The IT Revolution and the Stock Market: Evidence.” *American Economic Association* (Papers and Proceedings) 89, no. 2 (May 1999): 1203-20.

[18] Hall, Robert E. “The Stock Market and Capital Accumulation.” *American Economic Review* 91, no. 5 (December 2001): 1185-1202.

[19] Hobijn, Bart and Boyan Jovanovic. “The IT Revolution and the Stock Market: Evidence.” *American Economic Review* 91, no. 5 (December 2001): 1203-20.

[20] Jovanovic, Boyan and Peter L. Rousseau. “General-Purpose Technologies.” *Handbook of Economic Growth* 1B, Ch. 18 North-Holland (2006): 1181-1224.

[21] Jovanovic, Boyan and Peter L. Rousseau. “Vintage Organization Capital.” Working Paper No. 8166, National Bureau of Economic Research, March 2001.

[22] Kendrick, John, *Productivity Trends in the United States* (Princeton: Princeton University Press, 1961).

[23] Klepper, Steven and Peter Thompson. “Intra-Industry Spinoffs.” (June 2006)

[24] Lucas, Robert E. Jr.. "Adjustment Costs in the Theory of Supply." *Journal of Political Economy* 75, no. 4, Part 1 (August 1967): 321-334.

[25] Lucas, Robert E. Jr., and Edward C. Prescott. "Investment Under Uncertainty." *Econometrica* 39 (1971): 659-81.

[26] Lucas, Robert. "Asset Prices in an Exchange Economy." *Econometrica* 46, no. 6 (November 1978): 1429-55.

[27] Lucas, Robert. "On the Size Distribution of Business Firms." *Bell Journal of Economics* 9, no. 2 (Autumn 1978): 508-23. (1978a)

[28] Prescott, Edward and John Boyd. "Dynamic Coalitions: Engines of Growth." *AEA Papers and Proceedings*, (May 1987): 63-67.

[29] Sargent, Thomas. "Tobin's Q and the Rate of Investment in General Equilibrium." *Carnegie-Rochester Conference Series on Public Policy* 12 (1980): 107-54.

[30] Smithers, Andrew. "UK Stock Market: Value at End 2006." Report 285, Smithers & Co. Ltd., January 2007.

[31] U.S. Department of Commerce, Bureau of Economic Analysis, "National Income and Product Accounts." Washington, DC (October 2006).

[32] Wright, Steven. "Measures of Stock-Market Value and Returns for the U.S. Nonfinancial Corporate Sector, 1900–2002." *Review of Income and Wealth* 50, no. 4 (December 2004): 561-85.

5 Appendix

5.1 Growth in the deterministic case

Lucas (1988) solved explicitly for the optimal and the equilibrium rate of growth. We can do that too, but only for some parameter values. When q and z are constant, the rate of growth of capital and output is $g \equiv x + y - \delta$. We can solve for g with the help of the following result:

Proposition 2 *When (q, z) are constant, y still solves (5) and x satisfies the implicit function*

$$q = \beta (1 - \delta + x + y)^{-\sigma} (z - qx - h(y) + q[1 - \delta + x + y]) . \tag{20}$$

Proof. When q and z are constant, (6) becomes

$$q = \beta \left(\frac{C'}{C} \right)^{-\sigma} (z - qx - h(y) + q(1 - \delta + x + y)) .$$

But C must grow at the same rate as k, namely $x + y - \delta$, and this leads to (20). ■

Solving for g in a special case.—For the case in which $\sigma = 1$ and $h(y) = \frac{\gamma}{2} y^2$, we have $y = \frac{1}{\gamma} q$ and $h[y(q)] = \frac{1}{2\gamma} q^2$, and later on this Appendix shows that

$$x = \beta \frac{z}{q} - \frac{1}{2\gamma} (2 - \beta) q - (1 - \beta)(1 - \delta) \tag{21}$$

which is declining in q and increasing in z, β, and δ, and that

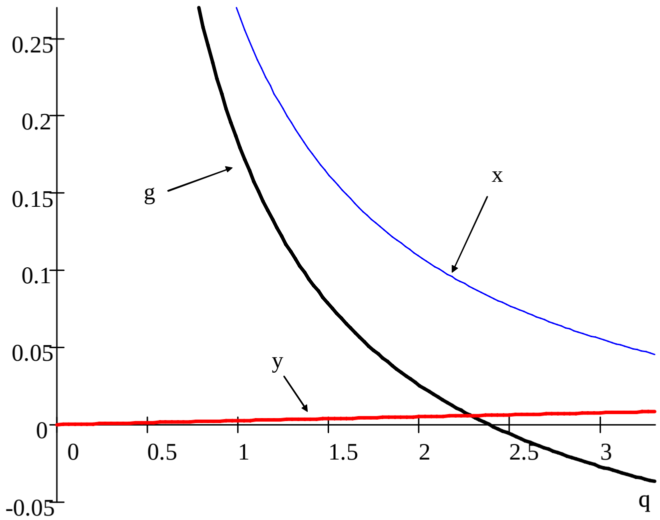
$$g = \beta \left(\frac{z}{q} + \frac{q}{2\gamma} \right) - (1 - \beta(1 - \delta)) \tag{22}$$

which is increasing in z and β, and decreasing in δ and in q, the latter being confined to the 'admissible' range in which $x > 0$.

Calibrated case.—If entrants investment is roughly one percent of GDP, then

$$\frac{Y}{zk} = \frac{y}{z} = \frac{q}{\gamma z} = 0.01. \tag{23}$$

Second, let the capital-output ratio be 3 so that $k/zk = 10$, implying that $z = 0.33$. The average value of q is 1.3. Then (23) implies that $\gamma = \frac{1.3}{(0.33)(0.01)} = 394$. We set $\beta = 0.95$, and $\delta = 0.10$ and obtain the following plot:



LONG-RUN GROWTH AND Q, γ=394, β=0.95, AND δ=0.10.

The growth rate is the rate is the black line, and we also plot the investment of entrants, q/γ as the red line and of incumbents $x = i - y = \delta + g - y$ as the blue line and plot the result in the Figure.

Derivation of (21).—When $\sigma = 1$ and $h(y) = \frac{\gamma}{2}y^2$, (20) reads

$$q = \beta\left(1 - \delta + x + \frac{1}{\gamma}q\right)^{-\sigma}\left(z - qx - \frac{1}{2\gamma}q^2 + q\left[1 - \delta + x + \frac{1}{\gamma}q\right]\right)$$

so that

$$1 - \delta + x + \frac{1}{\gamma}q = \frac{\beta}{q(1-\beta)}\left(z - qx - \frac{1}{2\gamma}q^2\right),$$

which, since $1 + \frac{\beta}{1-\beta} = \frac{1}{1-\beta}$, implies that

$$\begin{aligned} x &= \beta\left(\frac{z}{q} - \frac{1}{2\gamma}q\right) - (1-\beta)\left(1 - \delta + \frac{1}{\gamma}q\right) \\ &= \beta\frac{z}{q} + \beta\frac{1}{2\gamma}q - (1-\beta)(1-\delta) - \frac{1}{\gamma}q \end{aligned}$$

i.e., (21).

Derivation of (22).—Since $g = x + \frac{q}{\gamma} - \delta$,

$$\begin{aligned} g &= \frac{2q}{2\gamma} + \beta\frac{z}{q} - \frac{1}{2\gamma}(2-\beta)q - (1-\beta)(1-\delta) - \delta \\ &= \beta\frac{z}{q} + \frac{\beta}{2\gamma}q - (1-\beta) + \delta(1-\beta) - \delta, \end{aligned}$$

i.e., (22).

The 'admissible' range of qs for which $x > 0$.—Since x is decreasing in q, (21) states that the upper bound on q, call it q^u, solves the equation $x = 0$, i.e.,

$$\frac{1}{2\gamma}(2-\beta)\,q^2 + (1-\beta)(1-\delta)\,q - \beta z = 0,$$

which means that (letting $b = (1-\beta)(1-\delta)$),

$$q^u = \frac{\gamma}{2-\beta}\left(-b + \sqrt{b^2 + \frac{2(2-\beta)\,\beta z}{\gamma}}\right) < \frac{\gamma}{2-\beta}\sqrt{\frac{2(2-\beta)\,\beta z}{\gamma}} < \gamma\sqrt{\frac{2\beta z}{\gamma}} < \sqrt{2\gamma z},$$

where the first inequality follows because for any $n > 0$, $\sqrt{b^2+n} < b + \sqrt{n}$. Now differentiating in (22), this implies that

$$\frac{\partial g}{\partial q} = -\frac{z}{q^2} + \frac{1}{2\gamma} < -\frac{z}{2\gamma z} + \frac{1}{2\gamma} = 0.$$